Lecture 7: DH Parameters & Forward Kinematics

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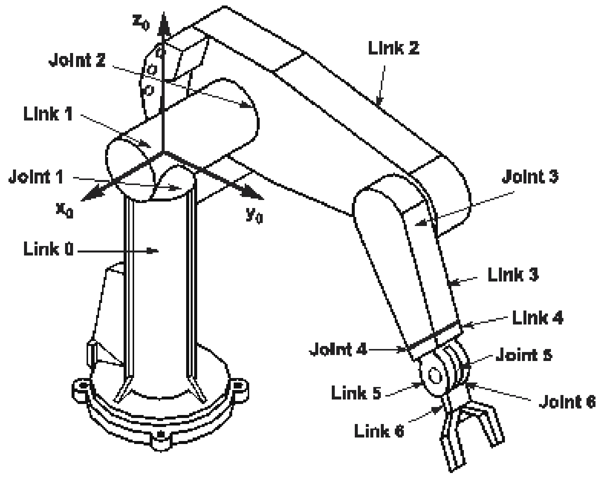

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What did we learn last time?

Recap of Lecture 6

What are we doing today?

1. CAD Modelling in Autodesk Inventor – *Guest Lecture by Aljaz Kramberger*
2. CAD Assemblies in Autodesk Inventor – *Guest Lecture by Aljaz Kramberger*
3. Introduction to Robotics & Recap of Linear Algebra and Mathematical Notation
4. Translations & Rotation Matrices
5. Other Representations for Orientation
6. Transformation Matrices
7. **DH Parameters & Forward Kinematics** ***(Today)***
8. Inverse Kinematics
9. Kinematic Simulation
10. Velocity Kinematics & the Jacobian Matrix
11. More about the Jacobian & Trajectory Generation
12. Manipulability, More on the Robotic Systems Toolbox

Topics for Today

Part I: Analytical Forward Kinematics

Part II: Parametrizing a Robot

- Recap of **robot joint types**
- **Axes** and **joints**
- Connect joints with **links**
- **DH Parameters**
- **Modified DH Parameters**

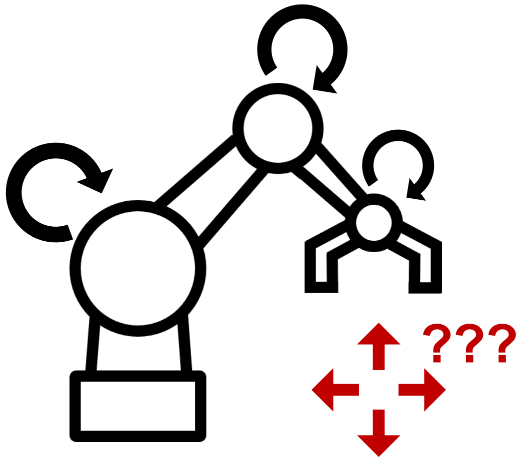
Part III: Forward Kinematics from DH Parameters

- **Single-link** transformations
- **Concatenated** link transformations
- Handling **offset frames**

Part I: Analytical Forward Kinematics

What is Forward Kinematics?

Forward kinematics describes how **motion of the joints** affects **motion of the robot end-effector**.



It can be solved by:

- Using a **geometric approach**.
- **Decomposing** the problem into a **series of transformation matrices**.

Forward Kinematics for 2R Planar Manipulator

Decomposing into Transformation Matrices

Objective: Obtain (x, y)

Given:

- Joint angles: q_1 and q_2
- Length of links: L_1 and L_2

By:

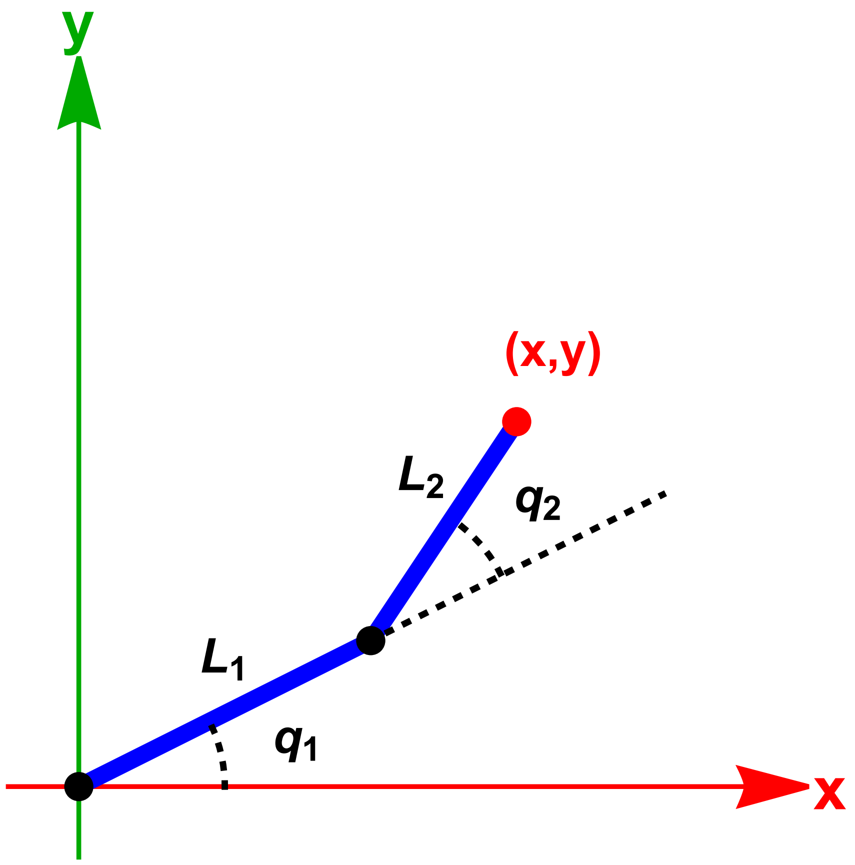
- Choosing appropriate frames
- Calculating their transformations

Hint:

- A 2D rotation matrix around the axis perpendicular to the plane is given by:

$$\boldsymbol{R}_z(\theta) = \begin{bmatrix} \cos q_1 & -\sin q_1 \\ \sin q_1 & \cos q_1 \end{bmatrix}$$

Exercise: Work <u>in small groups</u> (10 minutes)



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Forward Kinematics for 2R Planar Manipulator

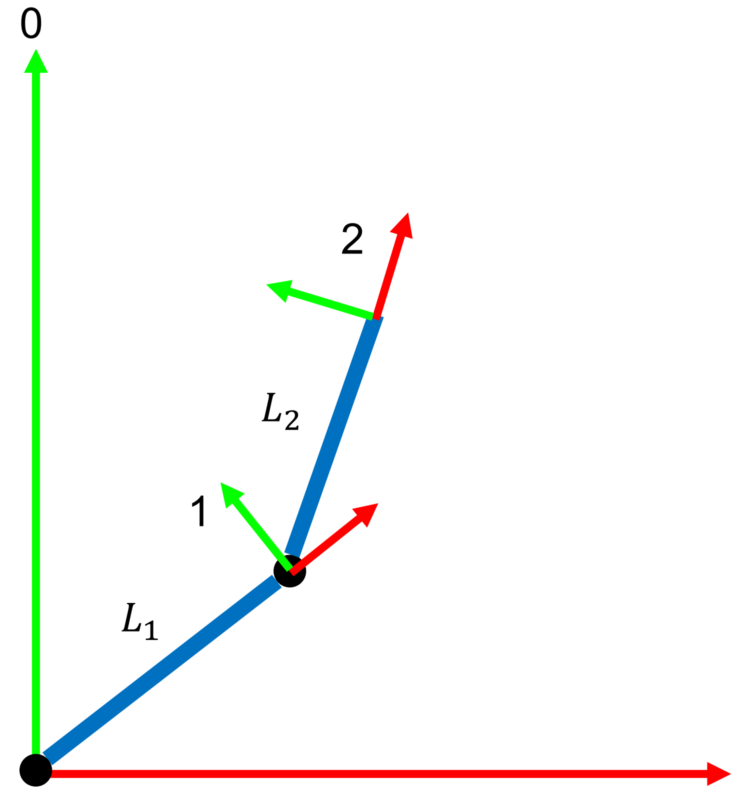
Decomposing into Transformation Matrices

One possible solution:

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} \cos q_1 & -\sin q_1 & L_1 \cdot \cos q_1 \\ \sin q_1 & \cos q_1 & L_1 \cdot \sin q_1 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} \cos q_2 & -\sin q_2 & L_2 \cdot \cos q_2 \\ \sin q_2 & \cos q_2 & L_2 \cdot \sin q_2 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{2}\boldsymbol{T} = {}^{0}_{1}\boldsymbol{T}\,{}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & L_1 \cdot \cos q_1 + L_2 \cdot \cos(q_1+q_2) \\ \sin(q_1+q_2) & \cos(q_1+q_2) & L_1 \cdot \sin q_1 + L_2 \cdot \sin(q_1+q_2) \\ 0 & 0 & 1 \end{bmatrix} $$



Forward Kinematics for 2R Planar Manipulator

Decomposing into Transformation Matrices

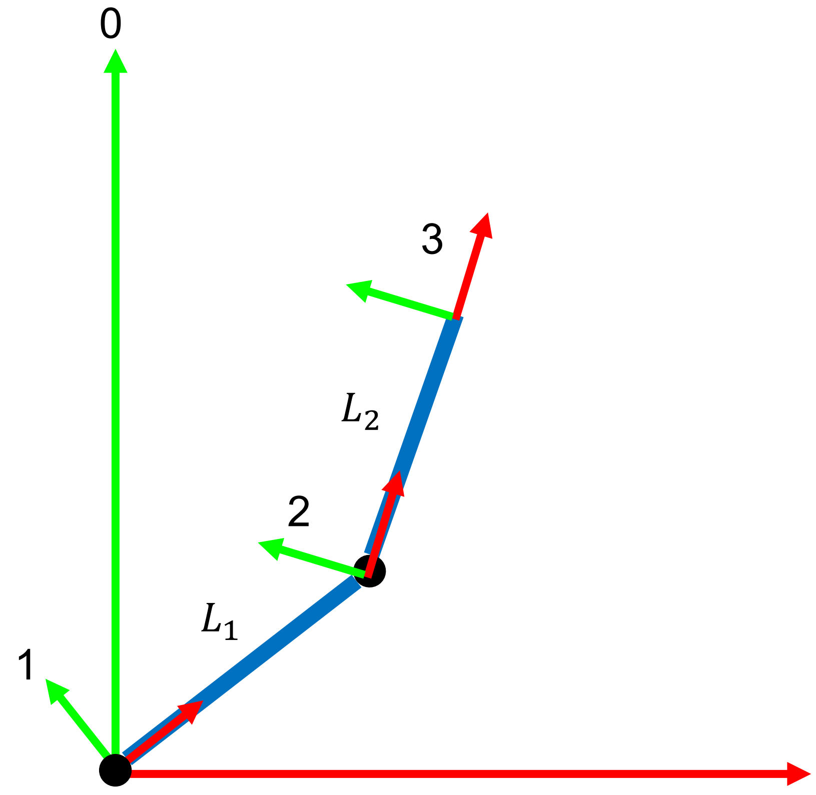
Another possible solution (more convenient):

$$ {}^{0}_{1}\boldsymbol{T} = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}\boldsymbol{T} = \begin{bmatrix} \cos q_2 & -\sin q_2 & L_1 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}\boldsymbol{T} = \begin{bmatrix} 1 & 0 & L_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{2}\boldsymbol{T} = {}^{0}_{1}\boldsymbol{T}\,{}^{1}_{2}\boldsymbol{T}\,{}^{2}_{3}\boldsymbol{T} = \begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & L_1 \cdot \cos q_1 + L_2 \cdot \cos(q_1+q_2) \\ \sin(q_1+q_2) & \cos(q_1+q_2) & L_1 \cdot \sin q_1 + L_2 \cdot \sin(q_1+q_2) \\ 0 & 0 & 1 \end{bmatrix} $$



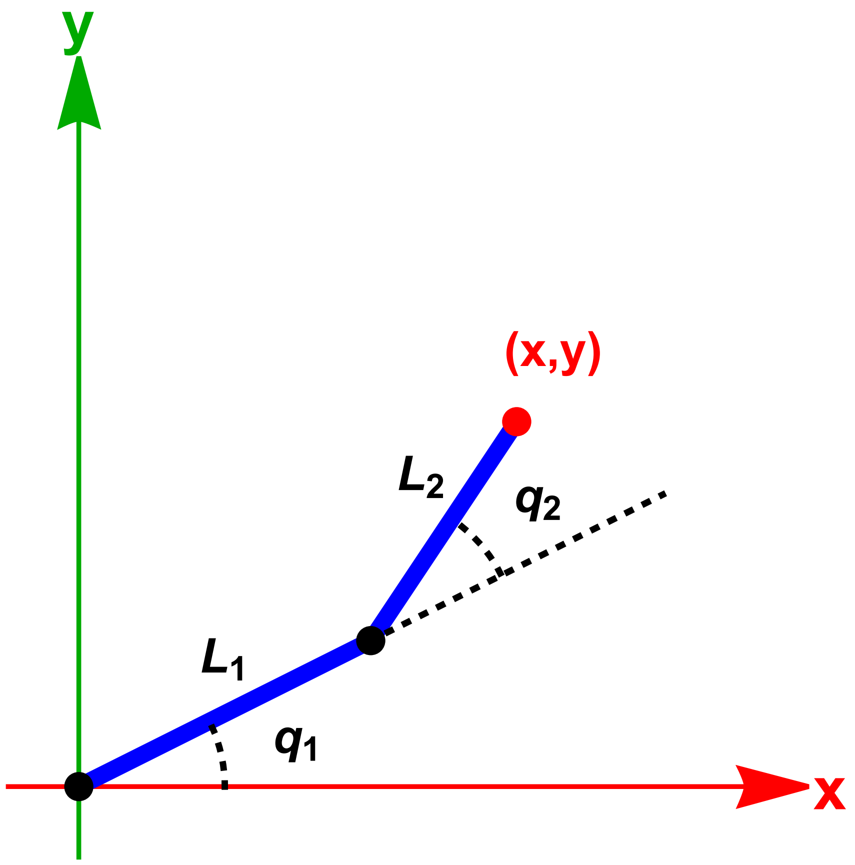
Forward Kinematics for 2R Planar Manipulator

Trigonometric Solution

Note that this robot is so simple, that we could also **calculate the solution using trigonometry:**

$$x = L_1 \cdot cos(q_1) + L_2 \cdot cos(q_1 + q_2)$$
$$y = L_1 \cdot sin(q_1) + L_2 \cdot sin(q_1 + q_2)$$

<u>**Real robots are rarely that simple.**</u>



Part II: Parametrizing a Robot

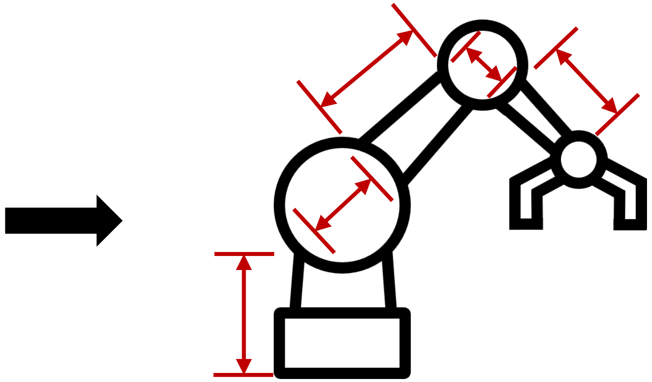
What do we want to achieve?

Forward Kinematics can always be calculated **analytically**.

The problem is that there is **no standard way**:
- **To place frames.**
- **To compactly describe a robot.**

Therefore, we want to:
Describe a robot in a **set of parameters**, so that its **structure is completely well-defined** with only these parameters.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3



What do we need to parametrize a robot?

We need a **standardized way** to:

1. Describe **how joints** are **placed on links**.

2. Describe how **coordinate frames** are **placed on joints**.

One such standard are **DH Parameters**, named after Denavit and Hartenberg [1].

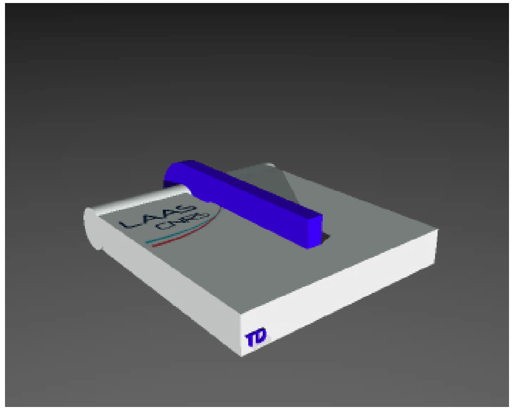
There are many variations, the most well-known being:

- Standard **DH Parameters** [1]
- **Modified DH** (used by J. Craig, i.e., in your book)

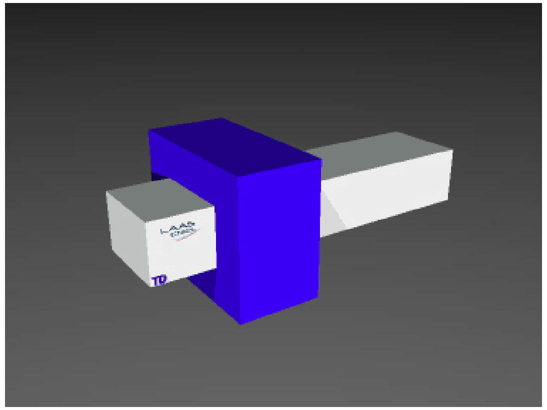
[1] J. Denavit, R. S. Hartenberg: A kinematic notation for lower pair mechanism based on matrices. In: Journal of Applied Mechanics, 1955.

Types of Robot Joints (Recap)

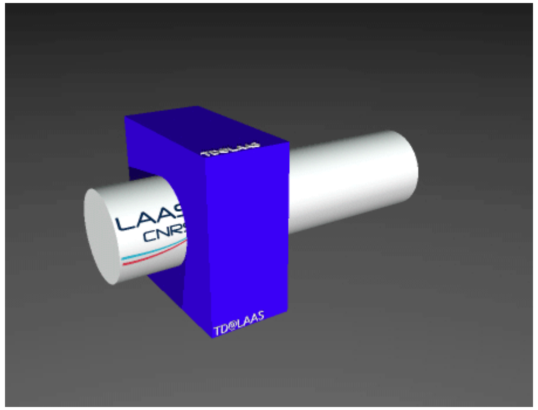
Joints can be of **different types** depending on **how they allow motion**:



Revolute

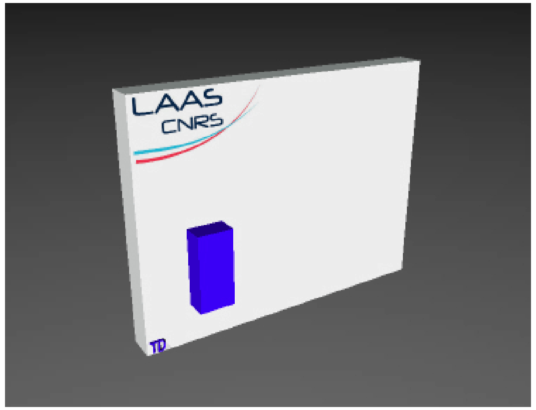


Prismatic



Cylindrical

Spherical



Planar

…and others

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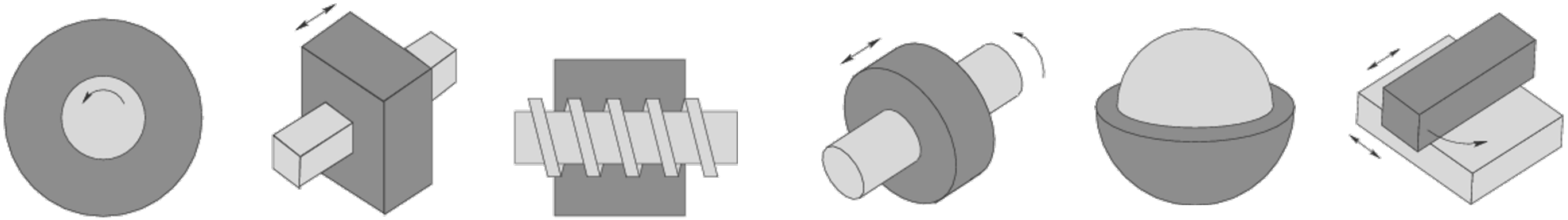
Source: https://gepettoweb.laas.fr/doc/stack-of-tasks/pinocchio/master/doxygen-html/md_doc_c-maths_c-joints.html

Types of Robot Joints (Recap)

Six possible so-called **lower pair** joint types result from two surfaces sliding over each other.

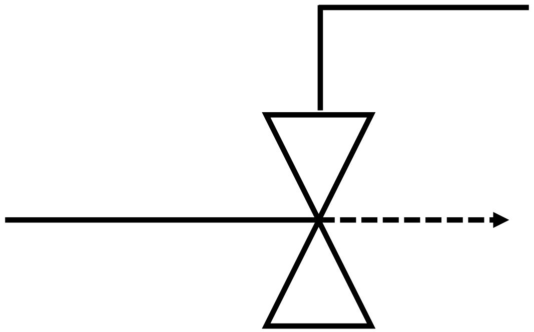


Revolute **Prismatic** **Screw** **Cylindrical** **Spherical** **Planar**

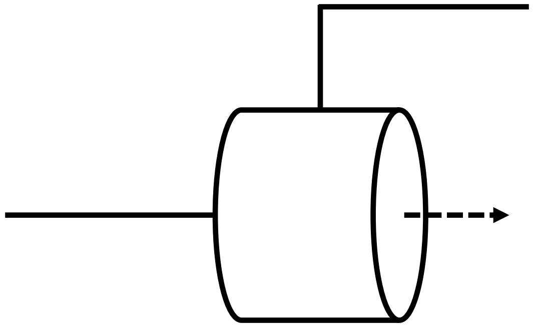
https://mechanical-engg.com/blogs/entry/949-types-of-kinematic-pairs/

Basic Joint Types

All other joints can be described as a **combination of:**



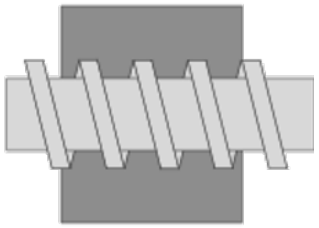
Prismatic joint – basic joint type to control the translation along 1 axis (e.g. push/pull cylinder).



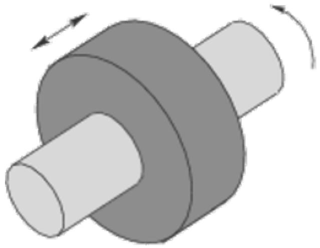
Revolute joint – basic joint type to control the rotation around 1 axis (e.g. turret or hinge).

In-class Exercise

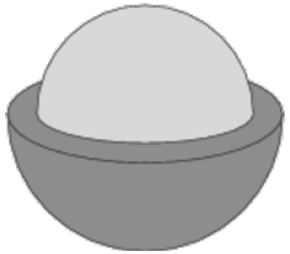
Describe the following **in terms of combinations** of **prismatic** and **revolute** joints:



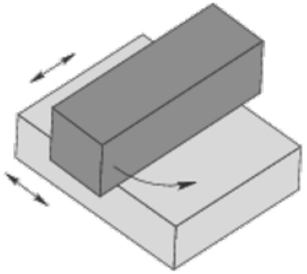
Screw



Cylindrical



Spherical



Planar

This is a bit of a joker… Describe instead what the input/output are.

Discuss with the class

https://mechanical-engg.com/blogs/entry/949-types-of-kinematic-pairs/

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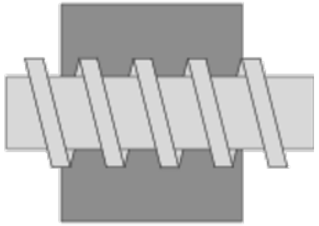
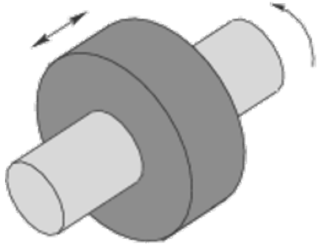
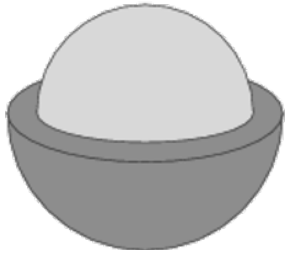
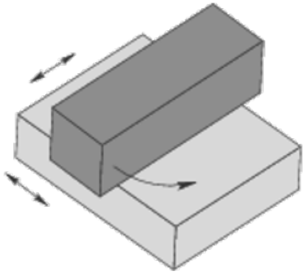
In-class Exercise: Solution

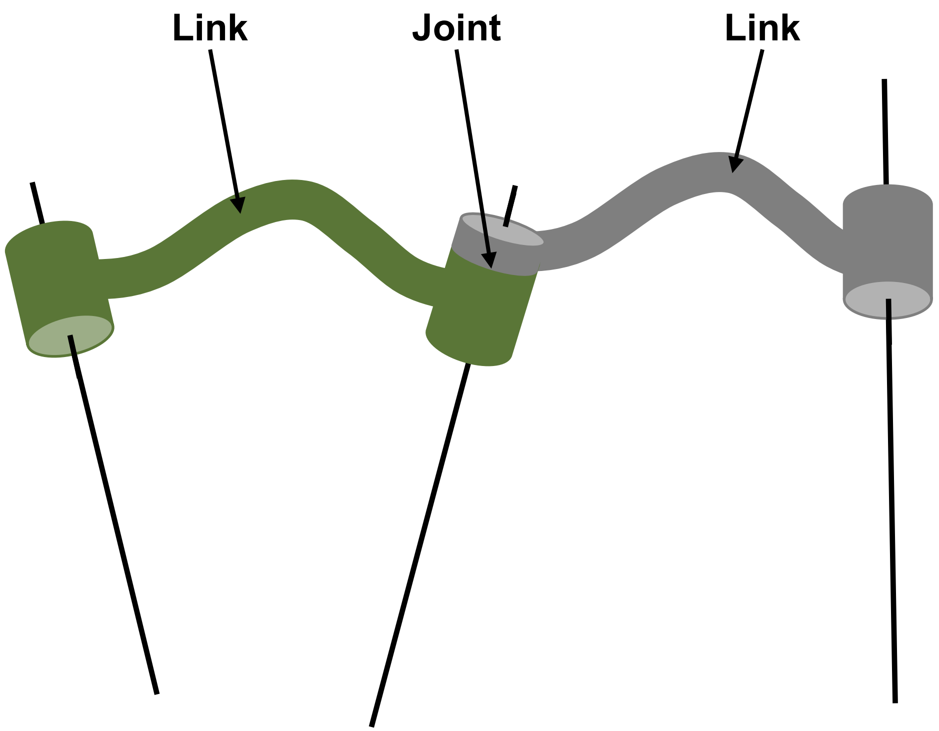
Describe the following **in terms of combinations** of **prismatic** and **revolute** joints:

			
Screw	**Cylindrical**	**Spherical**	**Planar**
Input: angle of screw turn Output: horizontal displacement	1 prismatic + 1 revolute	3 revolute	2 prismatic + 1 revoluate

https://mechanical-engg.com/blogs/entry/949-types-of-kinematic-pairs/

Joints: Connections between Links

Joints connect different **links**, allowing **relative motion** between them.

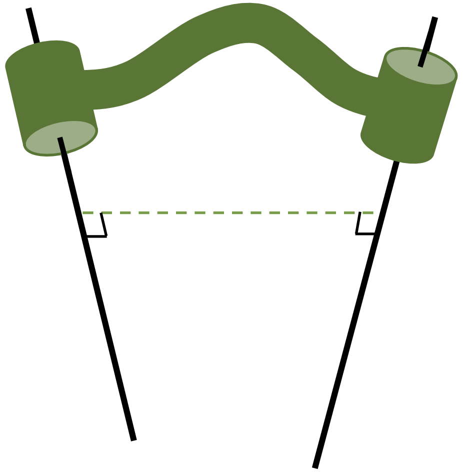


Links: Connections between Axes

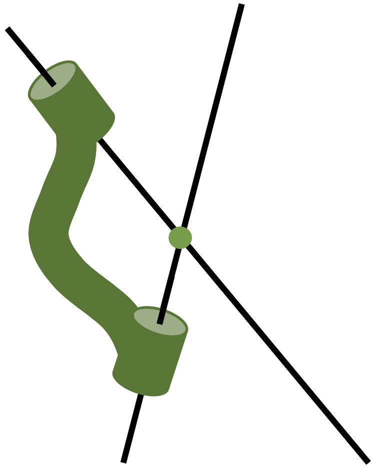
The kinematic function of a **link** is to maintain a **fixed relationship between two axes.**

(The axes may not be visible from the link's form.)

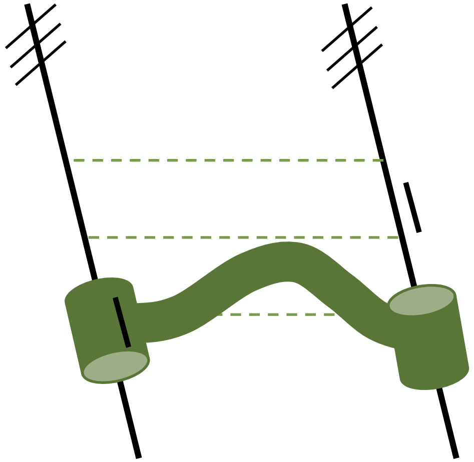
The **axes can have the following relationships**:



Skew axes



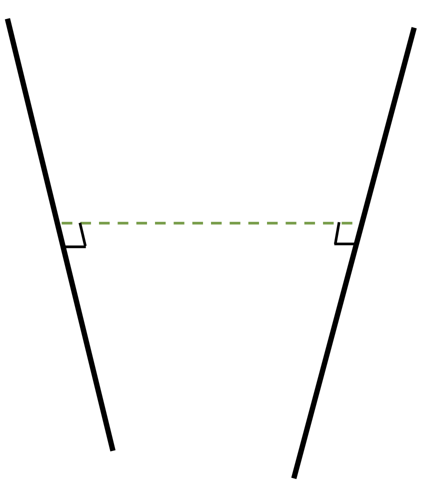
Intersecting axes



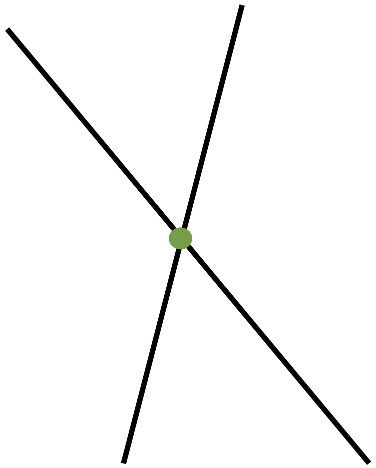
Parallel axes

Parts based on Kinematics Lecture Slides *by Christian Schlette, Professor, SDU Robotics*

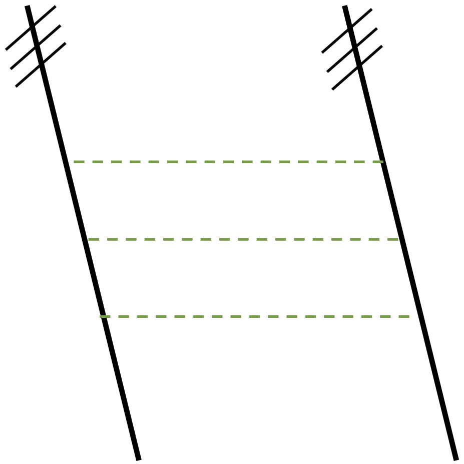
Two Axes in Space: Possible Relationships



Skew axes



Intersecting axes



Parallel axes

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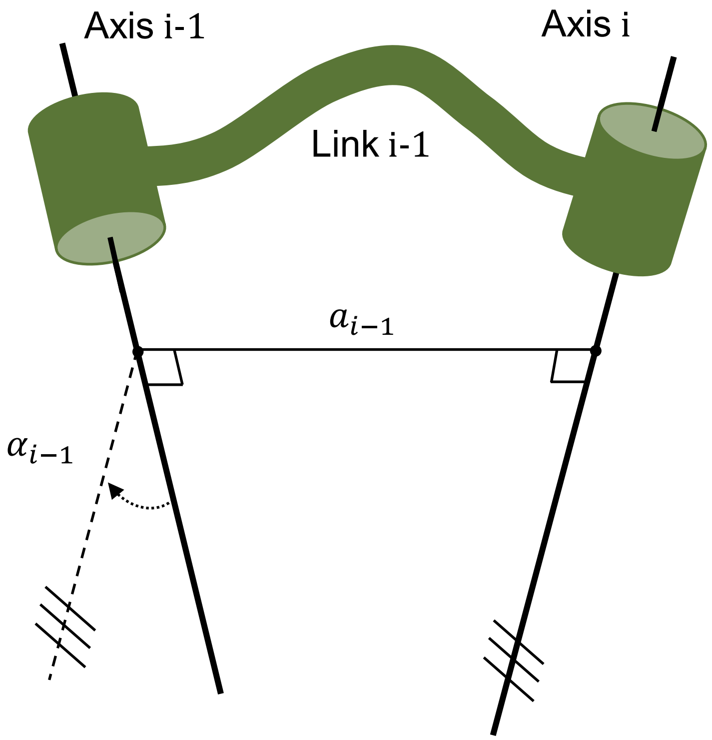
Parametrizing Links

All possible **relationships** (including intersecting and parallel axes) between two axes in space **can be described by two parameters**:

Link length a_{i-1}

Link twist α_{i-1}

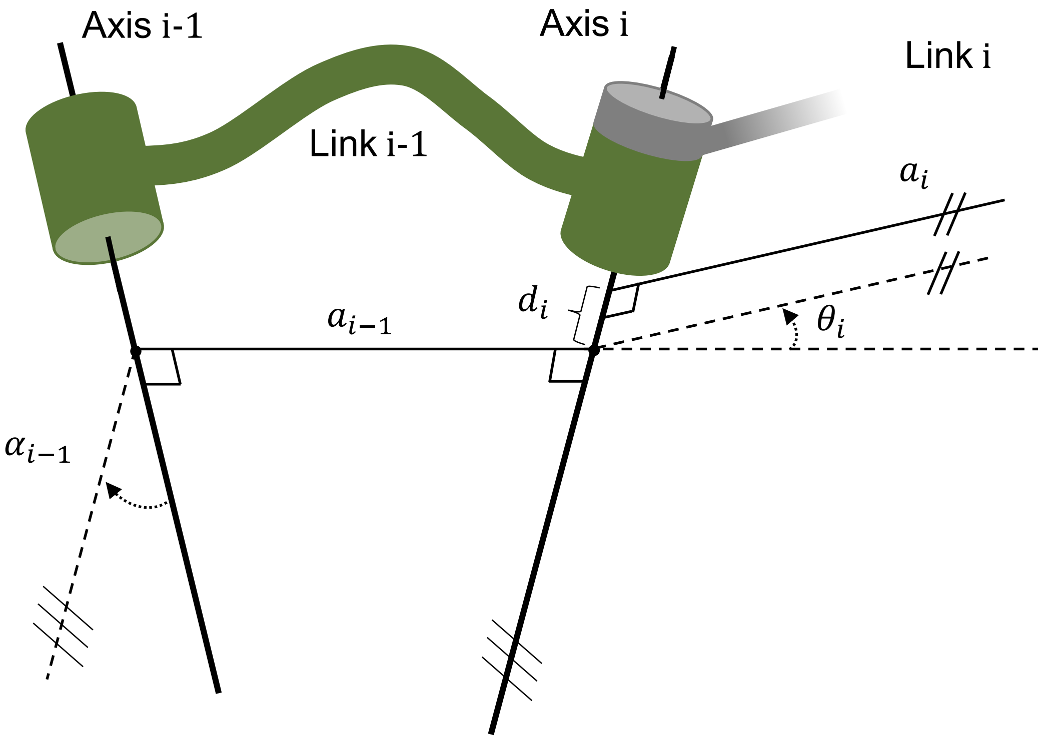


Parametrizing Connections between Links (Joints)

In addition, joints need a **joint variable** to describe their motion.

Link offset d_i
(Prismatic joint, $\theta_i = 0$)

Joint angle θ_i
(Revolute joint, $d_i = 0$)



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DH Parameters

Systematically describe kinematics consisting of **revolute** and/or **prismatic joints**.

Need only **four parameters** $(\alpha_{i-1}, a_{i-1}, d_i, \theta_i)$ to describe the relationships between succeeding joints.

i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
…				
n				

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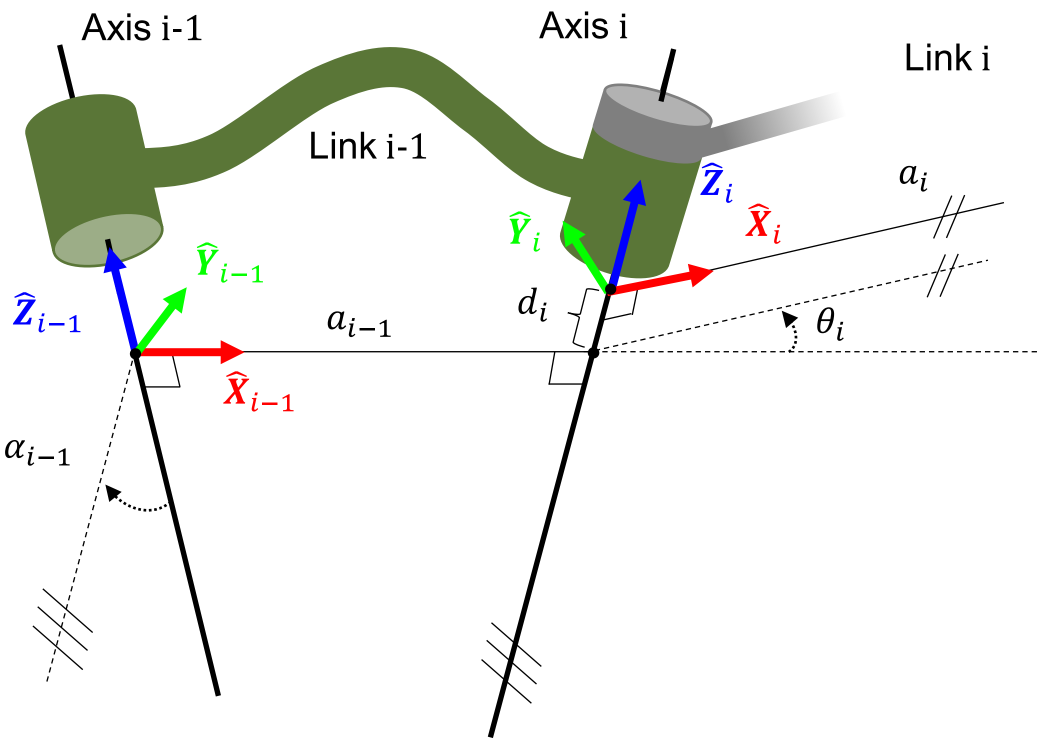
Frame Convention

Modified (Craig) DH

- The **origin** of frame $\{i\}$
  is located where a_i intersects joint axis i.
- The **Z-axis** of frame $\{i\}$
  is coincident with joint axis i.
- The **X-axis** of frame $\{i\}$
  points along a_i in the direction from joint i to joint $i+1$.
- The **Y-axis** of frame $\{i\}$
  follows in order to form a right-handed coordinate system.



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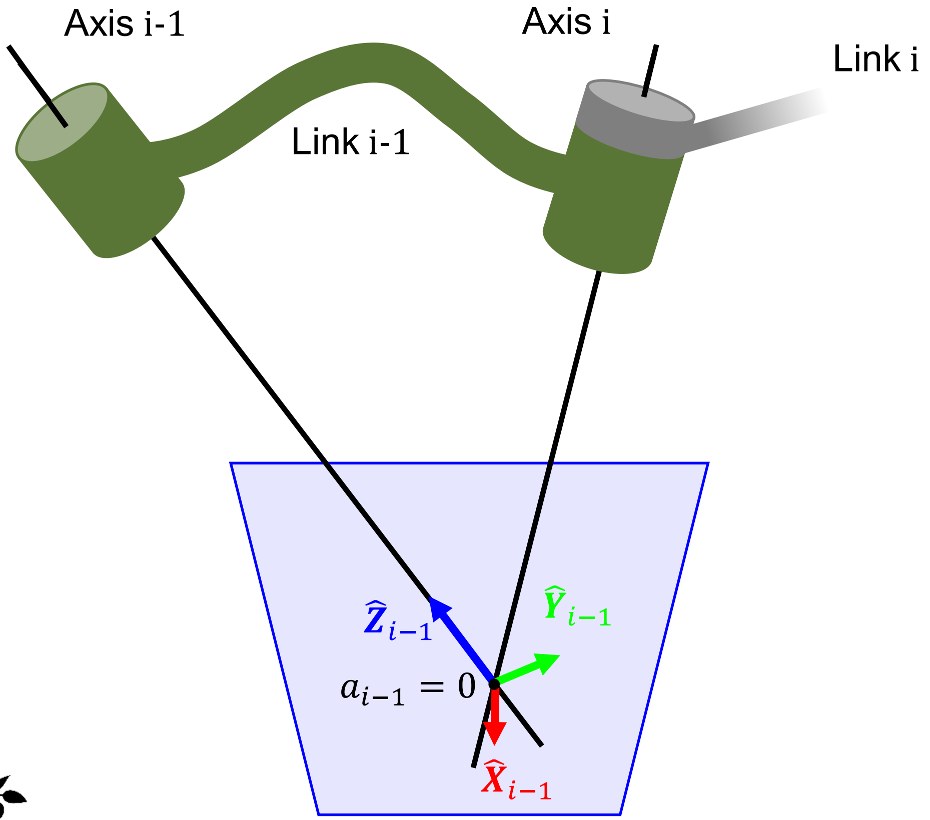
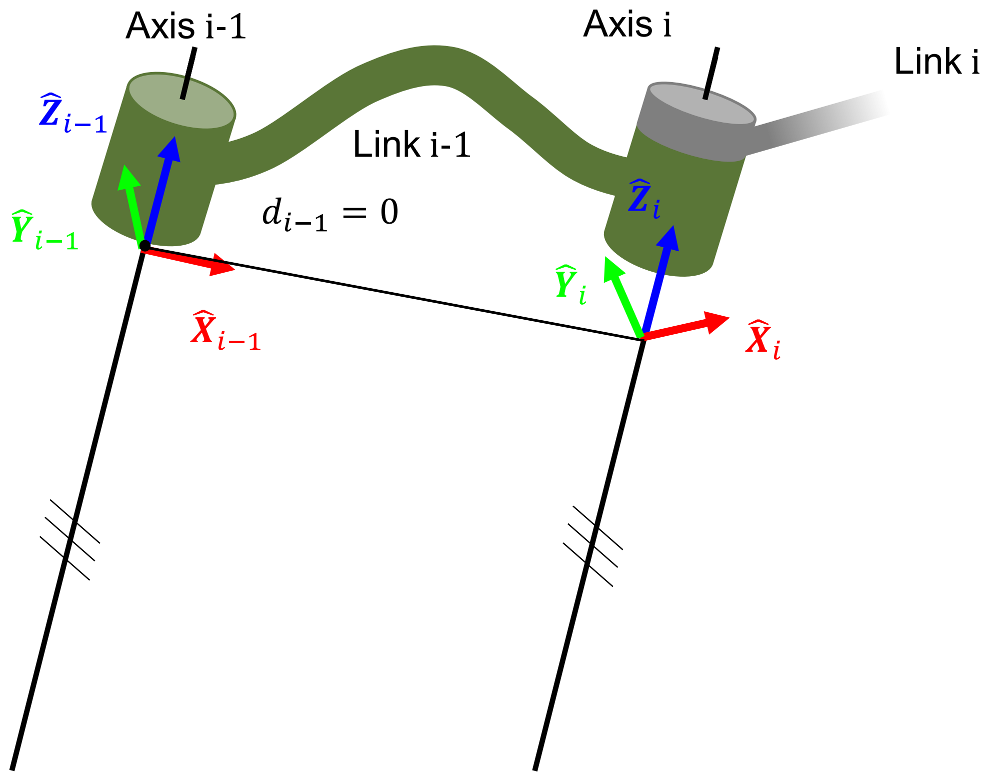
Frame Convention: Intersecting & Parallel Axes

Modified (Craig) DH

- **Intersecting axes** – Since $a_i = 0$, the X-axis of frame $\{i\}$ is defined to be the normal of the plane of $\hat{\boldsymbol{Z}}_i$ and $\hat{\boldsymbol{Z}}_{i+1}$ (arbitrary!)



- **Parallel axes** – Since a_i is not unique, the origin of frame $\{i\}$ is typically set to yield $d_i = 0$ (arbitrary!)

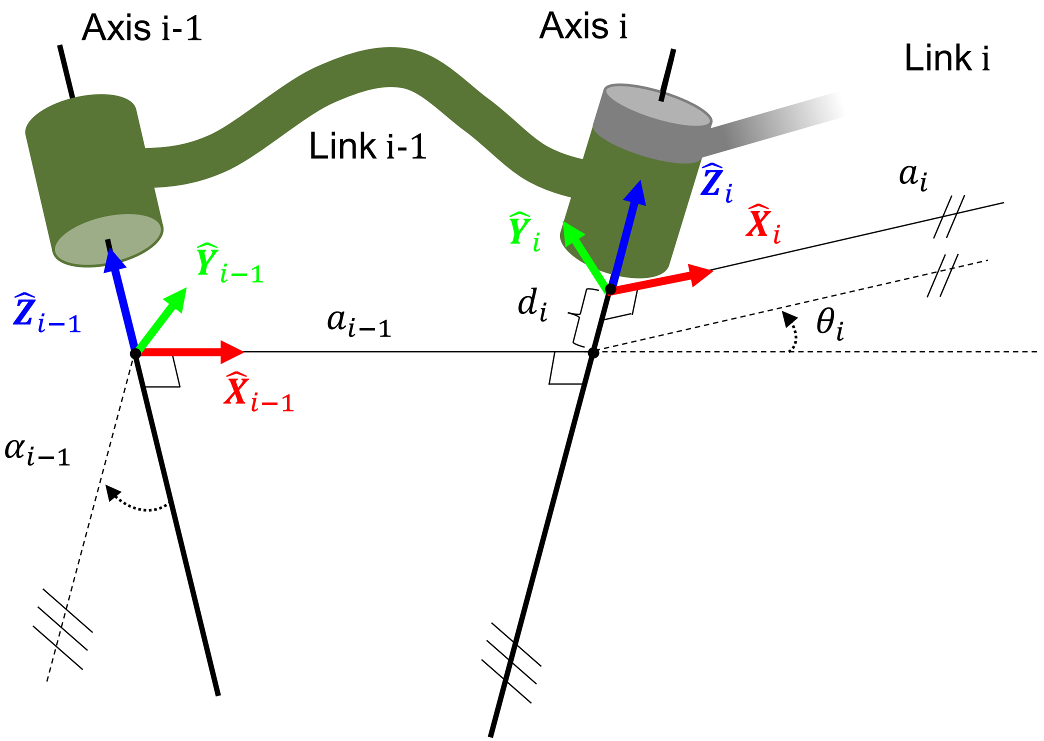


Frame Convention: First & Last Links

Modified (Craig) DH

- **First link –** A first frame $\{0\}$ is attached to the (fixed) base. Typically, frame $\{0\}$ is set to coincide with frame $\{1\}$, yielding $\alpha_0 = a_0 = 0$ and $d_1 = 0$ (joint 1 revolute), resp. $\theta_1 = 0$ (joint 1 prismatic) (arbitrary!)

- **Last link** – If joint n revolute, the direction of $\hat{\boldsymbol{X}}_N$ is set to align with $\hat{\boldsymbol{X}}_{N-1}$ and the origin of frame $\{N\}$ is set to yield $d_n = 0$. If joint n prismatic, the direction of $\hat{\boldsymbol{X}}_N$ is set to yield $\theta_n = 0$ and the origin of frame $\{N\}$ is set to the intersection of $\hat{\boldsymbol{X}}_{N-1}$ and joint axis n when $d_n = 0$.



DH Parameters: Procedure

1. **Identify** the **joint axes** and consider them as infinite lines.

2. **Identify** the **common perpendicular** between them or the **point of intersection**.

3. **Assign** the origins, **Z-axes** and **X-axes** of frames $\{i\}$ according to frame convention.

4. **Assign the Y-axes** of frames $\{i\}$ to yield **right-handed coordinate systems**.

5. **Assign frame {0}** and **frame {N}** according to convention.

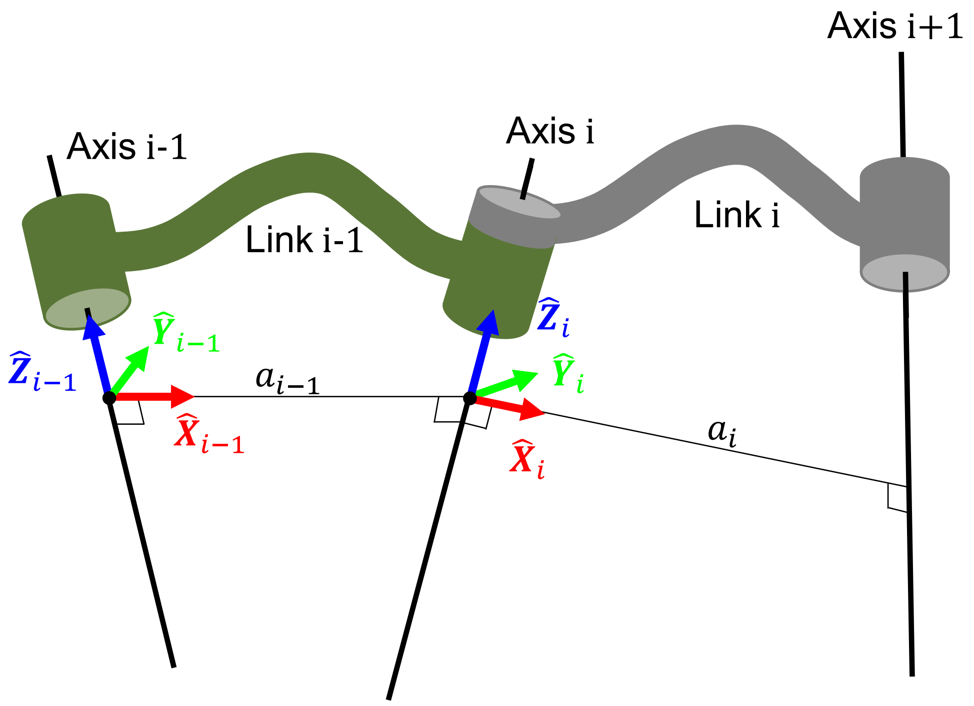
6. For each pair of frames $\{i\}$ and $\{i+1\}$, **identify the four DH parameters**:

- $\boldsymbol{\alpha_i}$ – The angle from $\hat{\boldsymbol{Z}}_i$ to $\hat{\boldsymbol{Z}}_{i+1}$ measured about $\hat{\boldsymbol{X}}_i$
- $\boldsymbol{a_i}$ – The distance from $\hat{\boldsymbol{Z}}_i$ to $\hat{\boldsymbol{Z}}_{i+1}$ measured along $\hat{\boldsymbol{X}}_i$
- $\boldsymbol{d_i}$ – The distance from $\hat{\boldsymbol{X}}_{i-1}$ to $\hat{\boldsymbol{X}}_i$ measured along $\hat{\boldsymbol{Z}}_i$
- $\boldsymbol{\theta_i}$ – The angle from $\hat{\boldsymbol{X}}_{i-1}$ to $\hat{\boldsymbol{X}}_i$ measured about $\hat{\boldsymbol{Z}}_i$

DH Parameters: Modified (Craig) vs Standard

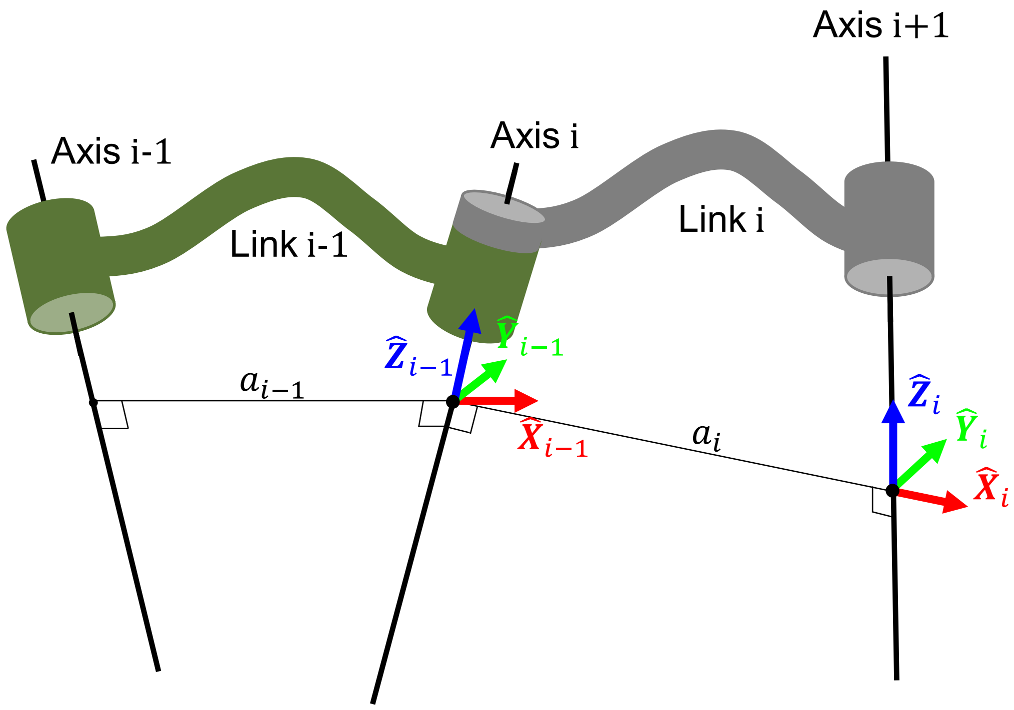
Modified (Craig)

- **Frame indices match** the **axes** they are placed on.
- x**-axis aligned** with the perpendicular to the **next frame**.



Standard

- **Frame indices** one count **behind** the **axes** they are placed on.
- x**-axis aligned** with the perpendicular to the **preceeding frame**.



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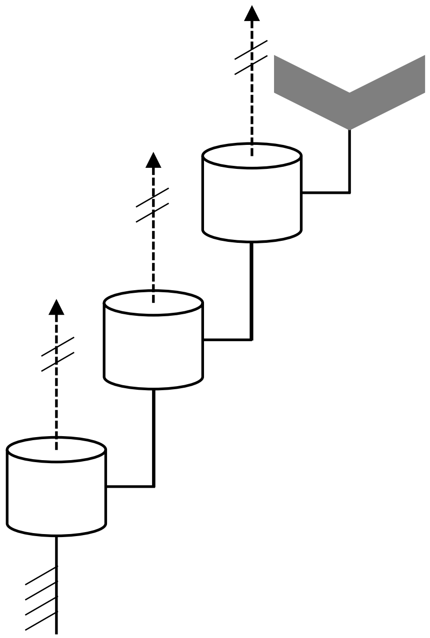
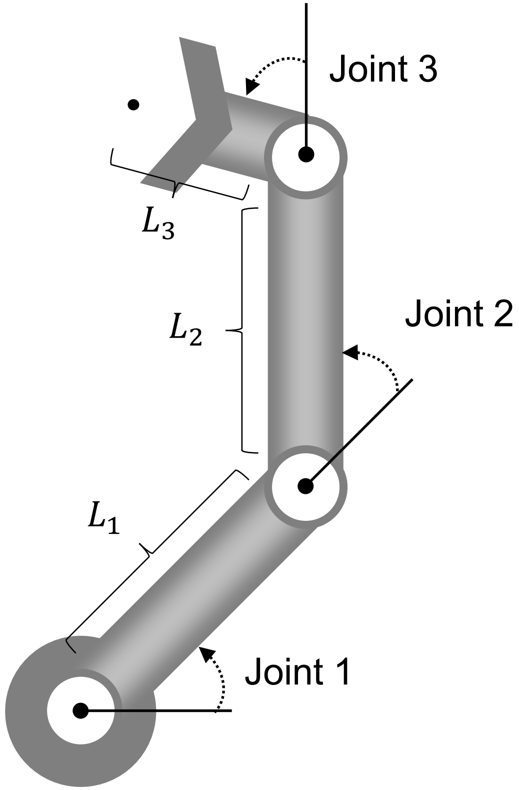
Three-link Planar Arm (RRR)

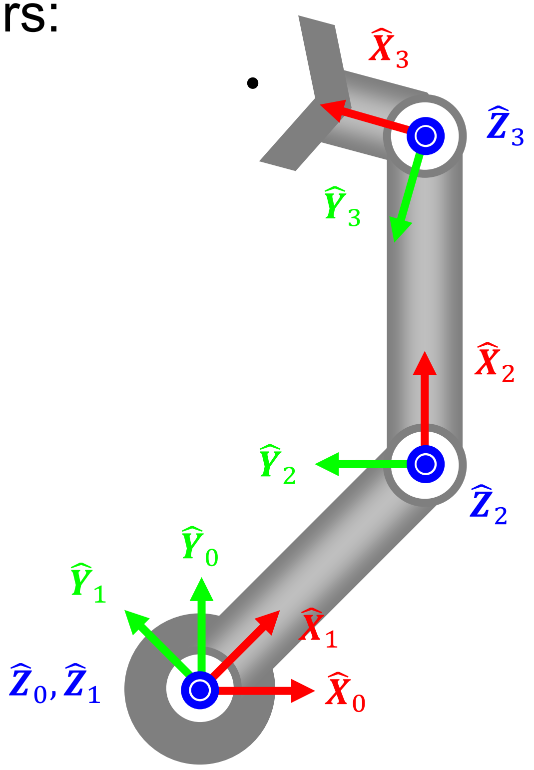




Exercise 1 (Individual, 15 min.)

Follow the procedure to identify the (Modified) DH parameters:

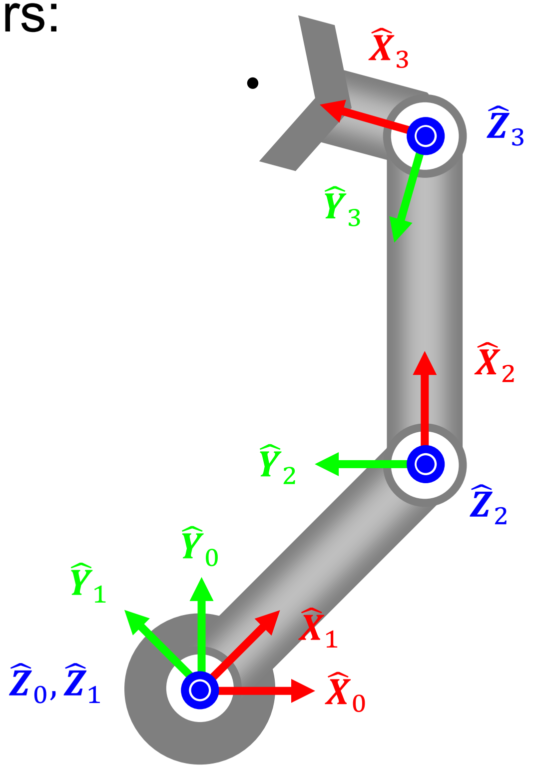
i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				
3				



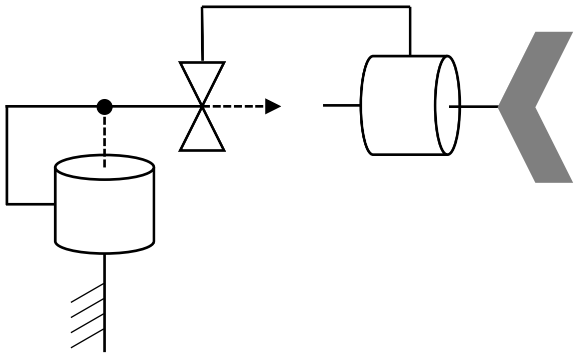
Exercise 1 (Individual, 15 min.)

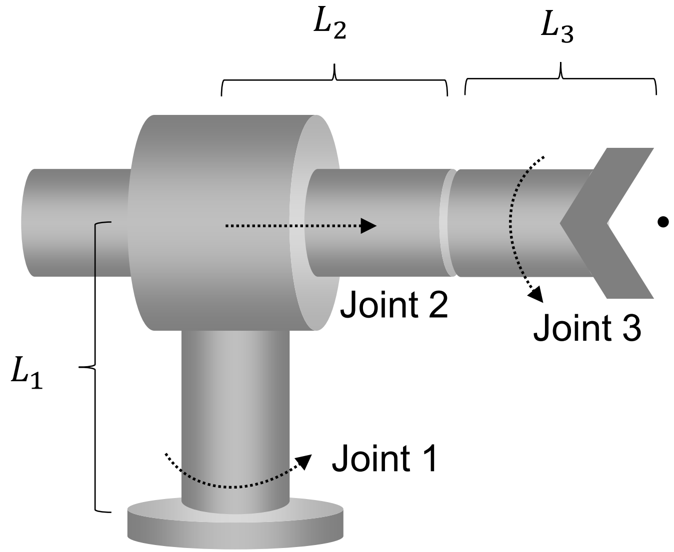
Follow the procedure to identify the (Modified) DH parameters:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3



Cylindrical Mechanism with Roll (RPR)

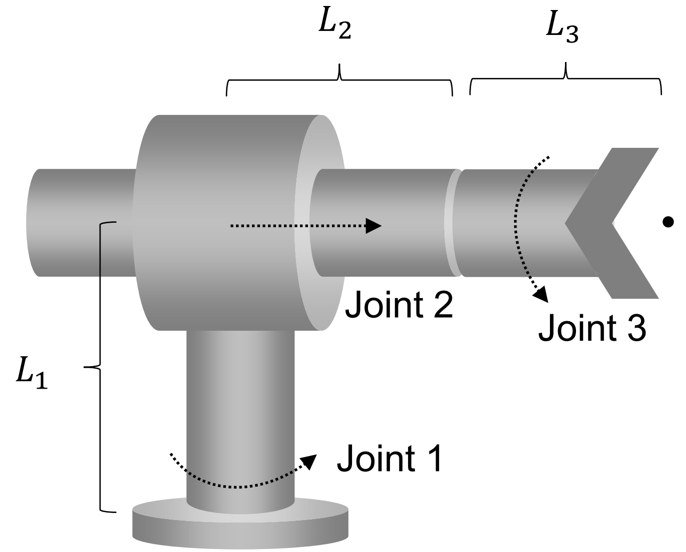




Cylindrical Mechanism with Roll (RPR)

Follow the procedure to identify the (Modified) DH parameters:

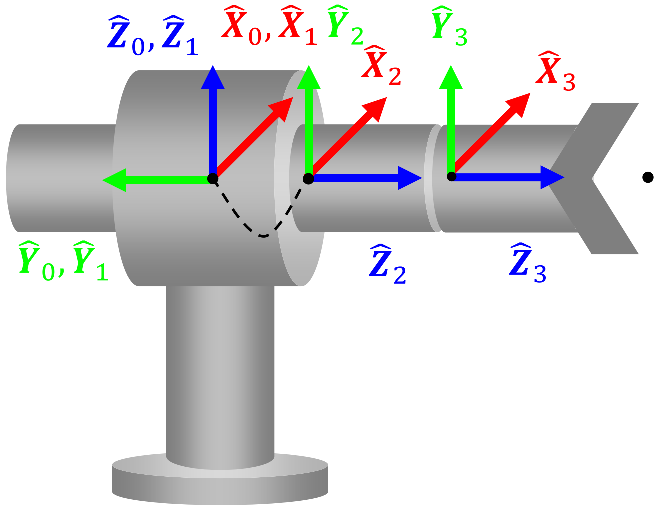
i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				
3				



Cylindrical Mechanism with Roll (RPR)

Follow the procedure to identify the (Modified) DH parameters:

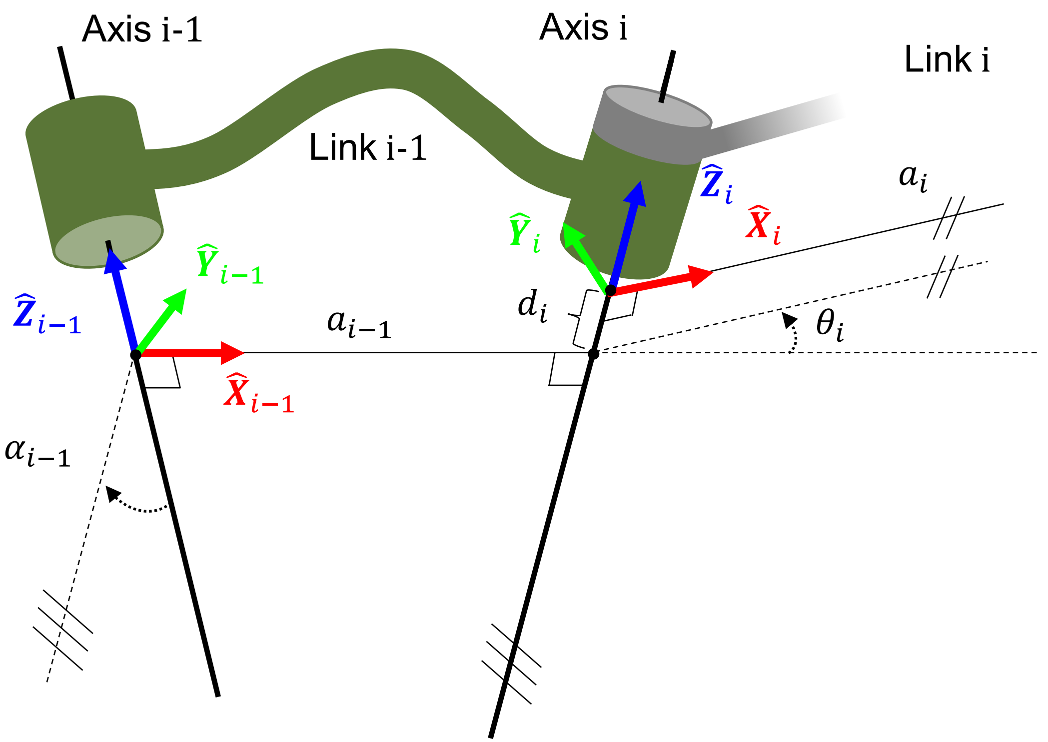
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3



Part III: Forward Kinematics from (Modified) DH Parameters

In-class Exercise (10 min.)

Derive a series of transformations to transform between frame {i-1} and frame {i}.



(*Hint: Use four transformations, each corresponding to one of the Modified DH Parameters*)

(*Hint: The order is rotation, translation, translation, rotation **and follows the order of the parameters***)

Single link transformation (Modified DH)

Step-wise transformation

Depart at frame {i-1}.

Rotate by α_{i-1} about $\hat{\boldsymbol{X}}_{i-1}$

$$\Longrightarrow \boldsymbol{R}_X(\alpha_{i-1})$$

Translate along $\hat{\boldsymbol{X}}_{i-1}$ about the distance a_{i-1}

$$\Longrightarrow \boldsymbol{D}_X(a_{i-1})$$

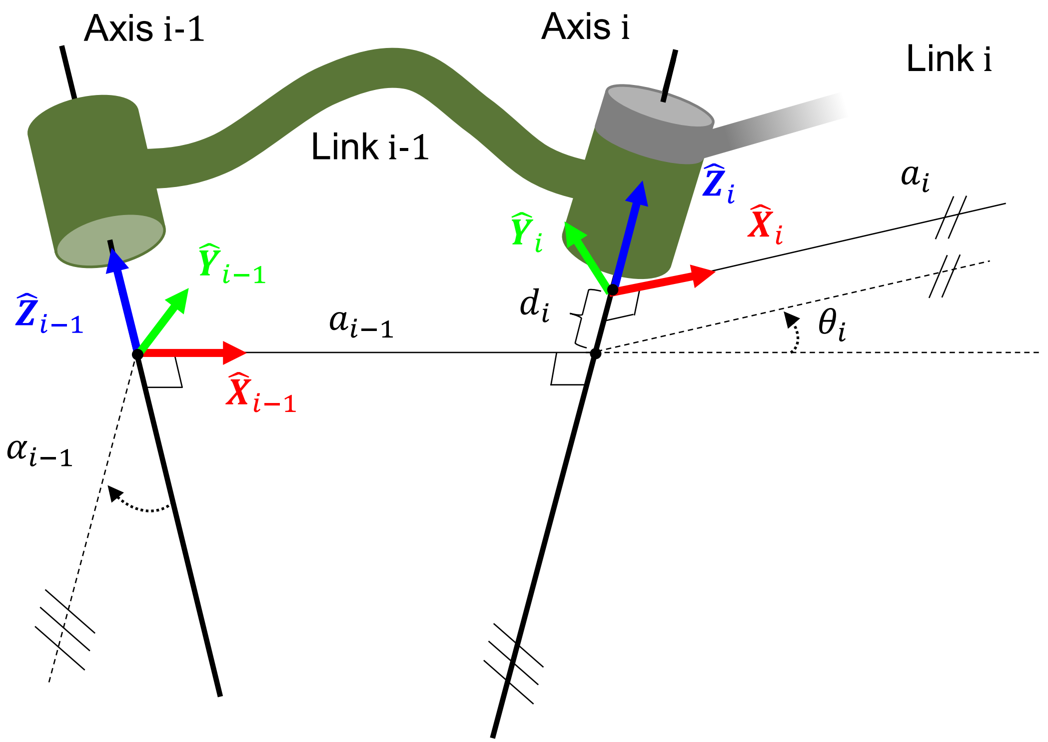
Translate along $\hat{\boldsymbol{Z}}_i$ about the distance d_i

$$\Longrightarrow \boldsymbol{D}_Z(d_i)$$

Rotate by θ_i about $\hat{\boldsymbol{Z}}_i$

$$\Longrightarrow \mathbf{R}_Z(\theta_i)$$

Arrive at frame $\{i\}$.



Single link transformation (Modified DH)

Combined transformation

These variables are always fixed — This is fixed for a revolute joint — This is fixed for a prismatic joint

$$ {}^{i-1}_{i}\boldsymbol{T} = \boldsymbol{R}_X(\alpha_{i-1})\boldsymbol{D}_X(a_{i-1})\boldsymbol{D}_Z(d_i)\boldsymbol{R}_Z(\theta_i) $$

$$ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Concatenated link transformations

Forward kinematics from DH parameters

If our robot has $\boldsymbol{N}$ **joints** we calculate all **single-joint forward transformations**: ${}^{i-1}_{i}\boldsymbol{T}$ for $i = 1, \dots, N$ **as in the previous slide**.

We can then **concatenate the transformations**:

$$ {}^{0}_{N}\boldsymbol{T} = {}^{0}_{1}\boldsymbol{T}\ {}^{1}_{2}\boldsymbol{T}\ {}^{2}_{3}\boldsymbol{T}\ \dots\ {}^{N-1}_{N}\boldsymbol{T} $$

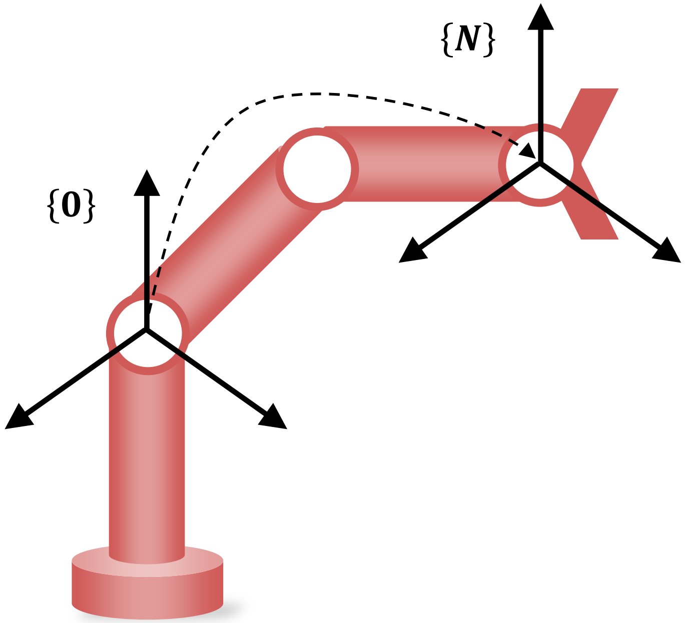
Each transformation depends on one variable, either:

- θ_i if revolute

or

- d_i if prismatic

Therefore, the **forward kinematics depends on $\boldsymbol{N}$ variables.**



SDU ROBOTICS

Offset frames

Base frame and Tool frame

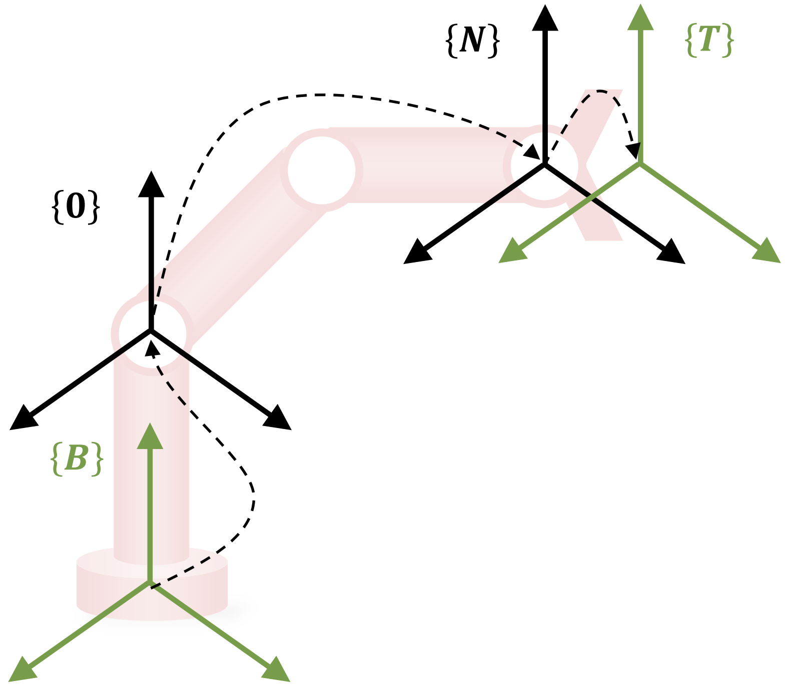
We are usually interested in calculating the **position of the tool in the base frame**.

DH parameters are missing these two frames:

- They start the first link (frame $\{0\}$),
  not the base.
- They end at the last link (frame $\{N\}$, called **Wrist frame**),
  not the tool.

We need to extend the forward kinematics by adding two **offset frames**, {B} and {T}:

$$ {}^{B}_{T}\boldsymbol{T} = {}^{B}_{0}\boldsymbol{T}\ {}^{0}_{N}\boldsymbol{T}\ {}^{N}_{T}\boldsymbol{T} $$



Recap: What have we discussed today

- All **lower pair joints** can be described as a **combination of revolute and prismatic joints**.
- **Joints connect together links**, allowing relative motion between them.
- **DH Parameters** allow us to **describe a link using four parameters:**
  - They describe the **relationship between two axes** that the link connects.
  - **Three** of these parameters **are fixed** and **one is a joint variable.**
- There are **multiple conventions for DH Parameters** (e.g. Craig's Modified notation and Standard DH).
  - They **differ on the placement and indexing** of the **frames**.
- Given the DH Parameters, the **forward transformation for a link** is a combination of **rotations and translations**.
- We can **concatenate forward link transformations** to derive the full **forward kinematics**.

Take home message:

DH Parameters allow us to **parameterize a robot** such that we can **describe the relationship between links and joints**.

Computing the **Forward Kinematics from the DH Parameters** can be done by **chaining** multiple **transformations** following **simple rules**.

Thank you for today.

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